The Paradox of Hard Work

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Abstract

Given substantial income effects on labor supply, empirical long-run increases in real output, consumption, and wages should have led to a significant reduction in work hours. Ultimately, this has not occurred. The aim of this paper is to shed light on why people are still working so hard, and what the implications of this paradox of hard work are for the economy as a whole. We develop a theory that focuses on the long-run macroeconomic consequences of trends in on-the-job (OTJ) utility. We find that given secular increases in OTJ utility, work hours will remain approximately constant over time even if the income effect of higher wages on labor supply exceeds the substitution effect. In addition, secular improvements in OTJ utility can be a substantial component of the welfare gains from ordinary technological progress. These two implications are connected by an identity: improvements in OTJ utility that have a significant effect on labor supply tend to have large welfare effects.

(JEL E24, J22, O30, O40)

1 Introduction

Over the last centuries there has been a dramatic world-wide increase in real output, consumption, and wages. Keynes predicted a large increase in leisure in his 1930 essay "Economic Possibilities for Our Grandchildren." Indeed, income effects on labor supply are substantial.[1]

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[1] See, for example, Kimball and Shapiro (2008).

However, the leisure boom predicted by Keynes has not taken place; instead, aggregate work hours have remained approximately constant relative to the behavior of other macroeconomic variables. This is highlighted in Figure 1, panels A through D, which shows the natural logarithm of consumption per population and work hours per population over the period 1960-2004 for the United States, Japan, the remainder of the G-7 countries, and a large set of European countries, relative to their 1960 values.[2] With the exception of relatively small declines in hours per population in European countries and relatively small increases in the US, the extent to which hours per population have remained relatively constant across countries, given ongoing and substantial increases in consumption per population, is striking. The objective of this paper is to understand why people are still working so hard, and what the implications of this *paradox of hard work* are for the economy as a whole.

There are, in principle, four alternative, although not mutually exclusive, explanations through which the paradox of hard work can be rationalized. The first is assuming that the elasticity of intertemporal substitution is large. However, empirical evidence suggests exactly the contrary. Hall (1988) finds this elasticity to be approximately zero, Basu and Kimball (2002) find that plausible values are less than 0.7, and Kimball, Sahm, and Shapiro (2011) find a value of approximately 0.08. The second is an increasing marginal-wage to consumption ratio. This can be the result of, for instance, a reduction in the progressivity of the tax system, an intensification of competition for promotions within firms, and increasing educational debts. The third relates to anything that keeps the marginal utility of consumption high. This can occur, for example, because of habit formation, both internal and external ("keeping up with the Joneses"), as well as the development of new goods. The fourth explanation relates to anything that serves to keep the marginal disutility of work low. This can be, for instance, the result of technological progress in household production, non-separability between consumption and leisure (King, Plosser, and Rebelo (1988), Basu and Kimball (2002)), and jobs getting nicer. Of the set of possible explanations, in this paper we focus attention on the impact of improvements in job utility both within the

[2] Data is at yearly frequency, and taken from the Penn World Tables and the Total Economy Database from The Groningen Growth and Development Centre. The "European Aggregate" consists of Austria, Belgium, Canada, Finland, Greece, Ireland, the Netherlands, Norway, Portugal, Spain, Sweden, and Switzerland.

context of separable and non-separable preferences. Economists have long understood that cross-sectional differences in job utility at a particular time give rise to compensating differentials. This paper develops a theory that focuses on a less-studied topic: understanding the long-run macroeconomic consequences of trends in on-the-job utility.[3]

Section 2 provides background for our research. In Section 3 we develop a benchmark model that allows us to study the interaction of *work hours* (which stands in for all aspects of the job that interfere with leisure and home production), *effort* (which stands in for all aspects of a job whose cost is in terms of proportionate changes in effective productive input from labor), *amenities* (which we define to be job characteristics whose cost is in terms of goods), and *drudgery* (which is a variable capturing everything else that matters for job utility). A novel result with regards to the Frisch elasticity of labor supply emerges, which is that this elasticity is decreasing in job utility. Therefore, the higher job utility is, the lower the volatility of work hours attributable to labor supply given temporary changes in the real wage.

Section 4 examines the determination of equilibrium, which is partly captured by way of two theoretical objects that result from explicitly accounting for on-the-job utility: *labor-earnings supply and labor-earnings demand*. Using our analytical framework, we show that ongoing declines in drudgery will, all else constant, eventually induce unambiguous increases in work hours. This stands in contrast to the long-run impact of ordinary technological progress, the effects of which can eventually result in income effects outweighing substitution effects. Overall, the analysis suggests that drudgery can be interpreted as an extended concept of technology.

Section 5 studies the implications of heterogeneity in production when considering differences in drudgery and technology in final goods producers, as well as across industries, and also in a setting of monopolistic competition. We show that firm- and industry-level job utility offerings play a critical role in determining the ability of firms and industries to endure across time given changes in economic conditions. Moreover, we argue that there are strong firm-level incentives for developing innovations that increase job utility.

[3] See, for example, Coulibaly (2006) for complementary research.

In Section 6 we examine the role of amenities. We show that the temporal evolution of amenities is inversely related to the temporal evolution of the marginal value of real wealth. As economies become richer, firms find it endogenously optimal to increase job utility via increases in amenities in order to (partially) mute the reduction in work hours that a lower marginal value of real wealth would otherwise tend to induce.

Our theory is explicitly related to the empirical trend(less) behavior of work hours in Section 7. We show that within our framework, given large increases in wealth, the extent to which work hours remain high, and for that matter, higher than expected, is a reflection of ongoing increases in on-the-job utility. In addition, we address welfare effects given changes in job utility in the alternative contexts of separable and non-separable preferences. We find that the welfare effects associated with the paradox of hard work can be substantial under either case. Finally, Section 8 concludes.

Our research contributes to the labor economics literature by developing a theoretical framework through which an intertemporal understanding of the primitives that determine the economy's available trade-offs between output, wages, and job utility can be attained. Our contribution to the macroeconomic literature is twofold. First, we show that secular improvements in on-the-job utility can induce work hours to remain approximately constant over time even if the income effect of higher wages on labor supply exceeds the substitution effect of higher wages. Thus, the paradox of hard work is not necessarily evidence that the elasticity of intertemporal substitution is large or non-separable preferences. Second, we show that secular improvements in on-the-job utility can themselves be a substantial component of the welfare gains from technological progress. These two implications are connected by an identity: improvements in on-the-job utility that have a significant effect on labor supply tend to have large welfare effects.

2 Background

Given the focus of this paper, a natural point of reference is the theory of compensating differences, which originates in the first ten chapters of Book I of "The Wealth of Nations"

(Smith (1776)). A standard modern reference on this topic is Rosen (1986). Denote the real wage by W, on-the-job utility by J, and output by Y. Figures 1 and 2 show, respectively, two well-known implications of the theory compensating differences. The solid line in Figure 2 is a wage/job-utility frontier reflecting that jobs that offer lower on-the-job utility will, in principle, compensate for this by offering higher real wages. Thus, all else equal, individuals face a tradeoff between these two variables. The solid line in Figure 3 implies that a similar tradeoff is faced by firms. As noted in Rosen (1986), firms can divert part of their productive resources towards improving on-the-job utility. Conditional on workers' individual preferences and firms' idiosyncratic costs of on-the-job utility in terms of output, each of these economic actors optimize by choosing a feasible point on the (J, W) and (Y, J) planes, respectively.

In Figures 1 and 2, all else equal, simultaneous increases in output and the real wage are consistent with movements along the solid curves. In this case, as indicated by the accompanying arrows, movements from points a to b and c to d, are consistent with decreases in on-the-job utility. This will result in a decrease in labor hours conditional on job utility being positively related to the amount of hours individuals desire to spend at work, which would further enhance the impact of strong income effects. Indeed, as argued in Kimball and Shapiro (2008), income effects on labor supply are substantial. Therefore, in the present context, relatively trendless labor hours require, in principle, ongoing increases in job utility in order to offset strong income effects. This involves secular northeastern shifts in the wage/job-utility and job-utility/output frontiers as exemplified by the dashed lines shown in Figures 1 and 2. In particular, as the economy's choice set expands, optimal choices should involve moving to points such as a' and d'.

Of course, northeastern shifts in the wage/job-utility and job-utility/output frontiers could in principle be the result of ordinary technological progress. The theory we develop focuses particular attention on understanding the foundations and secular implications of changes in the economy's choice set. As we show, choice-set expansions owing to ordinary technological progress do not necessarily result in reoptimization that involves increases in job utility as required to offset income effects. Instead, declines in drudgery do. Likewise, increases in amenities are shown to be endogenously optimal firm-wise responses to declines in the marginal value of real wealth that would otherwise trigger substantial reductions in work hours. Along with our focus on the welfare implications of relatively trendless labor hours, our analysis results in a novel time-series understanding of the macroeconomic impact of changes in job utility, which is complementary to the long-standing microeconomic static framework of compensating-differences analysis.

3 The General Framework

The model is cast in continuous time. Throughout the paper we omit time indexes in order to avoid notational clutter. Since our focus is on the labor market, we assume the context of a small open economy in which agents can freely borrow and lend at the exogenously determined real interest rate r (equal to ρ, the rate at which all economic agents discount the future, in steady state).

3.1 Households

For simplicity, we begin our analysis by focusing on effort and drudgery. The treatment of amenities is deferred until further in the paper.[4] First, consider effort: several dimensions impact this variable. For instance, the intensity of a worker's concentration on a task while at his or her work station, the amount of time spent at the water cooler or in other forms of on-the-job leisure, time spent cleaning and beautifying the work place, time spent in office parties during work hours, morale building exercises, amount of time spent pursuing worker interests that have some productivity to the firm but would not be the boss's first priority, etc. Let $\mathcal{E}$ be a vector describing all such dimensions of what the average hour of work is like, including the fraction of time spent in each different activity at work. We assume $\mathcal{E}$ is determined optimally by firms, and for simplicity focus on perfect monitoring so that moral hazard problems are not an issue. Let D denote the drudgery level associated with work and $\mathcal{J} = \mathcal{J}(\mathcal{E}, D)$ be the function that maps $\mathcal{E}$ and D into the hourly utility associated with

[4] Understanding the role of amenities is straightforward once the implications of drudgery are clear.

being at work.

The maximized value of $\mathcal{J}$ can, in principle, take on any sign. Let

$$J(E,D) = \max_{\mathcal{E}}\{\mathcal{J}(\mathcal{E},D)\} \text{ such that } \Theta(\mathcal{E}) = E. \tag{1}$$

Above, Θ is a function mapping the vector $\mathcal{E}$ into the number E, and E gives effective productive input from an hour of labor before multiplication by labor-augmenting technology. We henceforth refer to E as effort per worker and $J(E,D)$ as the job utility function. We assume that $J \gtreqless 0$ is such that $J_D < 0$, and we allow for the possibility of job utility being increasing in effort at relatively small effort levels, while decreasing in effort at relatively high levels of effort.[5] Note that $J_D < 0$ implies that in (E,J) space a decrease in drudgery causes an upward shift in the job utility function. That is, lower drudgery results in higher job utility at any given effort level.

As an example of how to interpret J, consider two production techniques: 1 and 2. Suppose that production technique 1, $\mathcal{J}_1$, results in relatively higher job utility at lower effort levels, and production technique 2, $\mathcal{J}_2$, results in relatively higher job utility at higher effort levels. Then, as shown in Figure 4, in (E,J) space the job-utility function J is the upper envelope (bold) of these two techniques.

Let a representative household's utility be a function of consumption of the final good C, work hours H, effort E, and drudgery D. We assume that households are infinitely lived, consist of a representative worker, and seek to maximize

$$\int e^{-\rho t}\mathcal{U}dt = \int e^{-\rho t}(U(C) + \Phi(T-H) + H \cdot J(E,D))dt. \tag{2}$$

Above, ρ is the rate at which all economic agents discount the future, t denotes time, T is an individuals's total per-period time endowment, U represents consumption utility and is such that $U' > 0$ and $U'' < 0$, and Φ denotes utility from off-the-job leisure, satisfying $\Phi' > 0$ and $\Phi'' < 0$. In this additively separable case of $\mathcal{U}$, we normalize J and Φ so that

[5] We consider this to be the more intuitive case, although our results are unaltered by assuming that job utility is always decrasing in effort.

$\Phi'(T) = 0$.[6] Given this normalization, $J > 0$ means that a worker would be willing to spend at least some time on the job even if unpaid. On the other hand, $J < 0$ means that the worker would never do such job unless paid. We contrast the present framework with the non-separable case when we address welfare issues.

Consider a worker employed in a job characterized by drudgery D, effort demand E, and real wage payment W. The individual's utility maximization problem is, taking these variables along with the real interest rate r as given, to choose a path for consumption, assets M, and work-hours to maximize equation (2) subject to

$$\dot{M} = rM + \Pi + WH - C. \tag{3}$$

In the budget constraint the price of consumption has been normalized to 1, Π represents non-labor, non-interest income, and for any variable X, $\dot{X}$ refers to its change over time. The current-value Hamiltonian associated with the household's problem is therefore

$$\mathcal{H} = U(C) + (\Phi(T-H) + H \cdot J(E,D)) + \lambda(rM + \Pi + WH - C), \tag{4}$$

where λ is the costate variable giving the marginal value of real wealth in the household's dynamic control problem. The first-order condition for consumption implies that $U'(C) = \lambda$. Substituting the underlying expression for optimized consumption into the Hamiltonian we can state the Hamiltonian maximized over C as

$$\overline{\mathcal{H}} = \left[U\left(U'^{-1}(\lambda)\right) - \lambda(rM + \Pi - C)\right] + \left[\Phi(T-H) + H \cdot (\lambda W + J(E,D))\right]. \tag{5}$$

Maximizing $\mathcal{H}$ over C and H is equivalent to maximizing $\overline{\mathcal{H}}$ over H. Note that only the second term on the right-hand-side of equation (5) depends on H. Therefore, to study the household's labor-supply decision we can focus on the optimization subproblem

$$\max_{H} \Phi(T-H) + H \cdot B, \tag{6}$$

[6] See the appendix for further details on this normalization.

where

$$B = \lambda W + J(E, D) \tag{7}$$

represents *hourly (marginal) net job benefits.*[7] Note that B captures the utils per hour that an individual derives from on-the-job activities.

The individuals's optimization subproblem implies that for any $H > 0$ the first-order necessary condition for optimal per-worker labor hours satisfies

$$\Phi'(T - H) = B. \tag{8}$$

Hence, at the optimal level of hours per worker the marginal utility from off-the-job leisure is set equal to hourly net job benefits. As shown in Figure 5, it follows that $\Phi'(T - H)$ is the labor-hours supply function and the market clearing device for work hours is, in fact, marginal net job benefits B.

Proposition 1. *The Frisch elasticity of labor supply is decreasing in job utility.*

Proof. Consider once more the solution to the worker's optimization subproblem. Since work hours are a direct function of marginal net job benefits, we can write $d\log H = \bar{\eta} d\log B$. Given $B = \lambda W + J$, holding everything constant except wages $d\log B = \lambda dW/(\lambda W + J)$. Rearranging, it follows that $d\log B$ is equal to $d\log W/(1-\zeta)$, where $\zeta = -J/\lambda W$. Therefore,

$$d\log H = \bar{\eta} d\log B \implies d\log H/d\log W = \bar{\eta}/(1-\zeta) \tag{9}$$

is the Frisch elasticity of labor supply. □

Proposition 1 implies that the higher job utility is, the lower the volatility of work hours attributable to labor supply given temporary changes in the real wage. Moreover, note that $B = \lambda W + J$ implies that $B = \lambda(W(1-\zeta))$. Therefore, ζ can be interpreted as the fraction of the wage that is a compensating differential.

[7] This solution method is similar to the one used in Kimball and Shapiro (2008).

3.2 Firms

Consider a representative firm whose jobs are characterized by drudgery D. The firm's production function is

$$Y = K^{\alpha} (ZEHN)^{1-\alpha},$$

where $\alpha \in (0,1)$, Y is output, K is capital, Z is exogenous labor-augmenting technology, N denotes the number of workers, H is hours per worker, and E is effort per worker. In addition, let R denote the rental rate of capital, which is exogenous to the firm.[8]

For any output level $\bar{Y}$ a firm's cost minimization problem involves choosing capital K and total work hours HN to minimize $RK + W(HN)$ such that $K^{\alpha}(ZEHN)^{1-\alpha} = \bar{Y}$. The solution to this problem yields the cost function

$$\mathcal{C}(\omega, R, Y) = R^{\alpha}/((\alpha^{\alpha}(1-\alpha)^{1-\alpha})\omega^{1-\alpha}Y), \tag{10}$$

where $\omega = W/(ZE)$ is the *effective wage*.

Given equation (10), the remaining issue in solving the firm's problem involves minimizing the effective wage, which translates into the subproblem

$$\min_{W,\, E} \omega = W/(ZE) \tag{11}$$

such that

$$\lambda W + J(E, D) = B. \tag{12}$$

In solving this optimization subproblem we assume that firms take the marginal value of real wealth λ as given, as they do the rental rate of capital R and equilibrium hourly net job benefits B.[9] Combining equations (11) and (12), and rearranging yields

$$J = B - \lambda Z \omega E. \tag{13}$$

[8] We assume no adjustment costs, so that $R = r + \delta$, where δ is the capital depreciation rate.

[9] Intuitively, λ and B may differ across workers. For now, we assume the existence of a representative household, and address the issue of worker heterogeneity later in the paper.

In (E, J) space equation (13) traces out all effort and job-utility combinations that are consistent with any given effective wage. Hence, this equation represents a firm's isocost lines. Given B, the solution to the firm's optimization subproblem is implicitly captured by the isocost line that has the algebraically greatest feasible slope. Such feasibility is determined by the firm's job utility function, which captures all job utility and effort combinations that a firm is able to offer. As seen in Figure 6 $\omega'' > \omega > \omega'$ and ω is the firm's optimal effective wage: it can do better than ω'', and although ω' is preferred to ω, the former is not feasible given the firm's job utility function.

Note that he solution to the firm's subproblem occurs at a point of tangency between the firm's job utility function and one of its isocost lines.[10] Optimality is thus defined by

$$J_E = -\lambda Z \omega \implies J_E E = -\lambda W. \tag{14}$$

Since $\lambda, E > 0$, for positive wages it is an endogenous result from equation (14) that at the optimal choice of effort $J_E < 0$.[11]

4 Equilibrium

Assume all firms are producers of the final consumption good and price takers in the product market. Then, use of equation (10) implies that under perfect competition firms with positive output must have

$$1 = \left(R^{\alpha} / (\alpha^{\alpha} (1-\alpha)^{1-\alpha}) \right) \omega^{1-\alpha}. \tag{15}$$

Rearranging,

$$W/(ZE) = \left(\alpha^{\alpha} (1-\alpha)^{1-\alpha} / R^{\alpha} \right)^{1/(1-\alpha)}. \tag{16}$$

[10] Note that the tangency optimization method we use is robust to situations as shown in Figure 1. More generally, we have not needed to assume concavity of the firm's job utility function in order to solve its optimization subproblem.

[11] The solution methodology employed in solving a firm's optimization subproblem is the same regardless of the sign of the wage. For ease of exposition we henceforth restrict attention to cases under which the real wages associated with any given job are positive. Of course, the canonical example of a real wage equal to zero is volunteer work. Moreover, note that *Dude Ranches* are an interesting example of negative real wages.

Thus, from the firm's point of view, under perfect competition the effective wage is an exogenously determined constant.[12]

The economy's general equilibrium can be determined by way of two graphical tools. The first of these is shown in Figure 7, which extends the intuition from Figure 6 to the present case where, as far as a representative firm is concerned, the slope of an isocost line $-\lambda Z\omega$ is entirely exogenously determined. Since equilibrium requires that cost minimization takes place, optimality continues to be summarized by a point of tangency between the job utility function and an isocost line. The left panel of Figure 7 shows optimal effort requirements E and job utility J, which implicitly define the optimal real wage $W = \omega ZE$ and hourly net job benefits B. The right panel of Figure 7 shows the determination of work hours H.

What remains to be determined is the economy's marginal value of real wealth λ. In general equilibrium, our open-economy framework has $r = \rho$. In equilibrium $C = rM + \Pi + WH$. Given the household's first-order condition for consumption, this implies that

$$\lambda = U' \left(rM + \Pi + WH \right). \tag{17}$$

Since $U'(\cdot)$ is decreasing in C, equation (17) yields a negative relationship between λ and labor earnings WH, which we call the *demand for labor-earnings* $\left(LE^D\right)$ *function*.

Now, consider the determinants of the configuration shown in Figure 7, where λ was taken as given. Suppose that the marginal value of real wealth increases from λ to λ'. Then, as the left panel of Figure 8 shows, the firm's isocost lines become steeper (ω and Z remain fixed). The tangency condition summarizing optimality implies that B and E increase, while J decreases. The right panel of Figure 8 shows that the increase in B induces an increase in H. In addition, since ω cannot change but E increases, W must increase so that ω remains fixed. This implies a positive relationship between λ and labor earnings WH, which we call the supply of labor-earnings $\left(LE^S\right)$ function:

$$WH = Z\omega E \left(\lambda Z, D\right) \cdot H \left(B \left(\lambda Z, D\right)\right). \tag{18}$$

[12] Recall that we refer to W as the real wage and to ω as the effective wage.

Note that both E and B are increasing in the product λZ. As shown in Figure 9, demand and supply for labor earnings, equations (17) and (18), jointly determine the economy's level of λ and WH.

4.1 Changes in Technology and Drudgery

This section addresses the alternative implications of changes in labor-augmenting technology and drudgery. We contrast cases in which λ is and isn't held constant. Moreover, we highlight parallels between changes in drudgery and labor-augmenting technology that point to the former's importance as an extended component of firms' overall technology.

Proposition 2. *An increase in technology Z, holding the marginal value of real wealth constant, results in lower job utility, an increase in the real wage, and higher marginal net job benefits, effort, and work hours.*

Proof. See Figure 10, which portrays the λ held constant effects of. As shown in the figure's left panel, for given λ and ω firms' isocost lines become steeper. Moreover, the increase in the real wage owes to the fact that $W = \omega ZE$, ω is fixed, and ZE increases. □

Corollary 1. *Holding the marginal value of real wealth constant, the impact on labor productivity of an increase in technology Z is greater than proportional to the increase in Z.*

Corollary 1 follows from the fact that, as noted above, holding λ constant, an increase in Z induces an increase in E. Therefore, the model implies a channel through which shocks to labor-augmenting technology can be amplified in terms of their effect on productivity given short-run fluctuations in Z.

Proposition 3. *Allowing for adjustment in λ, the effects of an in technology Z on marginal net job benefits, work hours, job utility, and the real wage are ambiguous. However, labor earnings WH increase and λ decreases.*

Proof. Suppose technology increases from Z to $Z' > Z$, and consider once more Figure 9. For given λ both W and H increase, so labor-earnings supply shifts out. The outward shift in LE^S implies a decrease in equilibrium λ and an increase in equilibrium WH. Now, return to Figure 10. Note that a decrease in λ means that after all adjustments in λ take place, firms' isocost lines will be less steep than before adjustment in λ (the increase in Z, on its own, makes isocost lines steeper). The extent to which isocost lines become less steep than for λ held fixed ultimately depends on the magnitude of the change in λ. Thus, the final level of B, H, W, and J relative to their values prior to the change in Z is ambiguous. □

Regarding the unambiguous decrease in λ and increase in labor earnings WH highlighted by Proposition 3, note that it could well be the case that in the new equilibrium W is higher than before the change in Z, but H is lower. This would be a situation in which the income effect dominates the substitution effect.

Turning towards changes in drudgery, note that three possibilities emerge conditional on $J_{ED} = 0$, $J_{ED} > 0$, or $J_{ED} < 0$. The first of these means that changes in drudgery do not affect how taxing extra effort is, the second that less drudgery makes extra effort more taxing, and the last that lower drudgery makes increases in effort less taxing. We focus on $J_{ED} < 0$, since it is the most intuitively appealing possibility.

Proposition 4. *Consider a decrease in drudgery from D to $D' < D$, and assume $J_{ED} < 0$. The marginal value of real wealth held constant effects of this change are an increase in marginal net job benefits, effort, work hours, and the real wage. The effect on job utility is ambiguous.*

Proof. As shown in Figure 11, when drudgery decreases from D to D' the job-utility function shifts up and for given λ becomes less steep at every effort level. As depicted in Figure 11 the change in drudgery results in an increase in job utility. However, this need not always be the case. This is because for a sufficiently small upward shift in the job-utility function, it could be that the level of job utility remains constant or actually decreases. □

With regards to Proposition 4, note that if a decrease in drudgery actually induces a decrease in job utility, then for given λ the qualitative effects of a decrease in drudgery and an increase in labor-augmenting technology Z are identical. In addition, regardless of the change in job utility, when $J_{ED} < 0$ a decrease in drudgery induces an increase in optimal effort requirements. This results in an increase in hourly effective labor productivity.

Proposition 5. *Allowing for adjustment in λ,if $J_{ED} < 0$, the effects of a decrease in* *drudgery on marginal net job benefits, work hours, effort, the real wage, and job utility is ambiguous.*

Proof. As shown in Figure 11, holding λ fixed a decrease in drudgery, given $J_{ED} < 0$, results in an increase in both H and W. This means that the labor-earnings supply function shifts out, delivering a new long-run equilibrium value of λ that is lower and WH that is higher than before the decline in drudgery. Returning to Figure 11, lower λ makes isocost lines less steep. The extent to which isocost lines become less steep than for λ held fixed ultimately depends on the magnitude of the change in λ. Thus, the final level of B, H, E, W, and J relative to their values prior to the change in D is ambiguous. □

Corollary 2. *The ambiguity noted in Proposition 5 is limited. If after the change in D the new peak of the J curve lies above the original level of B, then any new tangency condition consistent with positive wages will necessarily deliver a new equilibrium value of B, and therefore H, that is higher than the original one.*

Highlighted through Corollary 2 is the fact that for positive wages, ongoing decreases in drudgery, regardless of the sign of J_{ED}, will eventually lead to increases in work hours. This is the result of decreases in D inducing upward shifts in the J curve, and stands in contrast to changes in labor-augmenting technology in which the ultimate change in work hours is always, in principle, ambiguous.[13]

4.2 Heterogeneity in the Labor Force

Given the relevance of marginal net job benefits in determining work hours and the importance of labor-augmenting technology in the firm's optimization subproblem, it is of particular interest to understand the implications of a labor force that is heterogeneous in wealth and productive capacity. To address this, let there be a continuum of agents inhabiting the economy, indexed by m, with differences in individual marginal values of real wealth λ_m and idiosyncratic productivity θ_m. Appropriately reindexed, a type m individual's opti-

[13]Given the analytical methodology developed above, it is straightforward to show that when $J_{ED} = 0$ a decrease in drudgery leads to a decrease in the marginal value of real wealth, effort, and the real wage, along with an increase in equilibrium work hours that ultimately induces an increase in the product WH. Moreover, in the less intuitive case $J_{ED} > 0$ the effects of a decrease in drudgery on all of the model's endogenous variables is entirely ambiguous.

mization problem is analogous to the representative agent case. Accordingly analogous are the solutions to an individual's utility maximization problem and optimization subproblem.

We consider the case in which workers are perfect substitutes in production. The firm's production function is therefore

$$Y = K^{\alpha} \left(Z \int \theta_m E_m H_m N_m dm \right)^{1-\alpha}. \tag{19}$$

Appropriately reindexed, cost minimization is parallel to the representative agent case. Then, for a given worker of type m the firm's optimization subproblem involves choosing the real wage it pays this worker ,W_m, and the corresponding effort requirement, E_m, to minimize

$$\omega = W_m / (\theta_m Z E_m) \tag{20}$$

such that

$$\lambda_m W_m + J\left(E_m, D\right) = B_m. \tag{21}$$

The firm takes as given the marginal value of real wealth of type-m workers λ_m, as well as their equilibrium marginal net job benefits B_m.

As shown in Figure 12 the intuition and solution methodology developed under a representative worker carries over to the present context of worker heterogeneity. Interestingly, note that from the firm's point of view what is relevant about worker types is the product $\lambda_m \theta_m$. Let this product denote a worker's *hungriness*. Then, we can class individuals into supra types M, which are any arbitrary worker types for which the product $\lambda\theta$ is equal to some value χ_M.

Under perfect competition in the product market it is straightforward to show that the equilibrium effective wage is once more determined exogenously by equation (16). Hence, within this context the firm is always indifferent regarding the employment of any given worker type. Nonetheless, across individuals there are differences in the associated isocost-line slopes $(-\chi_M Z\omega)$, meaning that the isocost lines associated with individuals characterized by higher values of Γ are steeper relative to those with lower values of Γ.

Proposition 6. *There is ambiguity regarding relative real wage differences across type-M individuals. However, hungrier individuals exert higher effort and receive lower job utility. Moreover, they receive higher marginal net job benefits and therefore work longer hours.*

Proof. Consider workers of type-M and -N such that $\chi_M > \chi_N$. As shown in the left panel of Figure 13, individuals characterized by less hungriness are predicted to exert lower effort, enjoy higher job utility, and receive lower hourly net job benefits than their counterparts with greater hungriness. As shown in the right-hand panel of Figure 13, workers with greater hungriness are predicted to work more hours. Relative real wages are in principle ambiguous since they depend on relative differences in idiosyncratic productivity as opposed to the product $\lambda_m \theta_m$ (see the definition of ω in the present context). □

It follows that relatively wealthy workers (lower λ) who are highly productive (high θ) can have relatively high hungriness and therefore be found to work relatively high hours at high effort levels. In addition, note that given constant returns to scale in production and mobile capital, different worker types can be thought of as being on their own individual islands. Therefore the comparative steady state analysis for a small open economy with one type of worker is exactly identical to the analysis of an economy with differences in workers as considered above. Thus, for ease of exposition, in what follows we revert to a representative worker framework.

5 Heterogeneity in Production

5.1 Differences in Final-Good Producers

Let there be a continuum of firms indexed by i, and assume that each firm is a producer of the final consumption good. We allow firms to differ in their labor-augmenting technology, drudgery levels, and job utility functions. All firms are perfectly competitive in the product market, and (economy-wide) marginal net job benefits continue to be the market-clearing device. Although the slope of firm i's isocost lines is given by $-\lambda Z_i \omega$, all of our earlier results carry over to the present context by straightforward reindexing. Moreover, some interesting new issues come into play. To make our points intuitively concise, we focus on a dual-firm framework.

Proposition 7. *Across firms, differences in drudgery can potentially countervail differences in technology, and vice versa.*

Proof. Consider firms 1 and 2 as shown in Figure 14, where $D_1 < D_2$, $Z_2 > Z_1$, and firms differ in their J curves. As depicted, firm 1 implicitly sets the economy's level of marginal net job benefits. Workers take the jobs with the highest B, so firm 2 is unable to attract workers and, therefore, operate. It is straightforward to see that for some lower value of D_2 (or higher Z_2) firm 2 could potentially offer the same level of B as firm 1, in which case both firms would be able to operate. Moreover, for even lower D_2 (or even higher Z_2) firm 2 could potentially be the one to implicitly establish economy-wide B, in which case firm 1 would be unable to attract workers. Analogous considerations follow even when firms have identical functional forms for their J curves. □

Proposition 7 highlights the fundamental importance of job utility for the (ongoing) existence of firms relative to the traditional concept of technology. In fact, drudgery can legitimately be viewed as a component of an expanded concept of overall technology. We defer the treatment of applicable versions of labor-earnings supply and demand until further below, where we consider the implications of industry-level differences. At this point, however, note that because in equilibrium B is the same across all employment opportunities, then in equilibrium individuals are willing to supply work hours to all firms that are able to operate. However, individuals will not necessarily be willing to spend the same amount of time working at any given job. As highlighted by Proposition 8, the more time an individual has to spend on overall work activates or the more exogenous wealth he or she is endowed with, the more time an individual will devote to jobs that offer greater job utility.

Proposition 8. *Consider firms 1 and 2. Suppose that firm i offers the job-utility/real wage bundle (J_i, W_i) and $J_2 > J_1$. Let ξ be the fraction of total work hours an individual devotes to working in firm 1. Then,* $\lambda = -\left(J_1 - J_2\right) / \left(W_1 - W_2\right)$, *and*

$$\xi = \frac{1}{W_1 - W_2}\left(\frac{U'^{-1}\left(\lambda\right) - rM - \Pi}{T - \Phi'^{-1}\left(B\right)} - W_2\right). \tag{22}$$

Proof. In appendix.

5.2 Industry-Level Differences

Suppose now that there is a continuum of industries indexed by i. Each industry produces a different type of good, but firms within industries are perfectly competitive. For ease of exposition, let there be a representative firm per industry. In analogous fashion to earlier analysis, denote industry i's labor-augmenting technology by Z_i. Moreover, let p_i be the relative price of the good produced by industry i. Worker-side optimization is just as before, and appropriately re-indexed the same is true of a firm's cost minimization problem.

The industry-level optimization subproblem is now to choose W_i and E_i to minimize the industry-level effective wage ω_i, which is equal to W_i/Z_iE_i. The relevant constraint is

$$\lambda W_i + J_i\left(E_i, D_i\right) = B_i, \tag{23}$$

where B_i are marginal net job benefits in industry i (in equilibrium, marginal net job benefits are equalized across all industries that operate). Analogously to the earlier development, combining the objective function and constraint yields the isocost line

$$J_i = B_i - \lambda Z_i \omega_i E_i.$$

Thus, in (E_i, J_i) space the solution to an industry's optimization subproblem again involves being on the isocost line that has the algebraically greatest feasible slope. What is different relative to the earlier analysis is that now, for industries with positive output the marginal cost of production must equal the relative price of an industry's output:

$$p_i = \left(R^\alpha / (\alpha^\alpha \left(1-\alpha\right)^{1-\alpha})\right) \omega_i^{1-\alpha}. \tag{24}$$

Rearranging,

$$\left(1-\alpha\right) \alpha^{\alpha/(1-\alpha)} / R^{\alpha/(1-\alpha)} = W_i \cdot p_i^{-1/(1-\alpha)} / \left(Z_i E_i\right) = \bar{\omega}, \tag{25}$$

where we have used the definition of ω_i. Therefore, from an industry's point of view it is now $\bar{\omega}$ that is an exogenously determined constant.

To fully appreciate the solution to an industry's optimization subproblem, note that an isocost line can be stated as

$$J_i = B_i - \lambda\bar{\omega}x_i, \tag{26}$$

where $x_i = Z_i p_i^{1/(1-\alpha)} E_i$. Furthermore,

$$J_i\left(E_i, D_i\right) = J_i\left(\delta x_i, D_i\right), \tag{27}$$

where $\delta = \left(Z_i p_i^{1/(1-\alpha)}\right)^{-1}$. Figure 15 shows the solution to an industry-level representative firm's optimization subproblem in $(\delta x_i, J_i)$ space. Since the slope of isocost lines, $-\lambda\bar{\omega}$, is the same across industries, then industry-level optimal operations and marginal net job benefits are determined by the point of tangency between a representative firm's isocost line and job utility function. Note that in Figure 15, the vertical distance between B_i and J_i is equal to $-\lambda W_i$, and the horizontal distance between $Z_i E_i p_i^{1/(1-\alpha)}$ and the origin is equal to $W_i/\bar{\omega}$.

Proposition 9. *Decreases in the marginal value of real wealth will tend to drive out industries offering relatively low levels of job utility per effort.*

Proof. Consider two industries, $i = 1, 2$ with job utility functions given by J_1 and J_2. As shown in Figure 16, for a low marginal value of real wealth such as λ' industry 1 is able to offer the highest marginal net job benefits, so industry 2 is unable to attract workers and, therefore, operate. For a higher marginal value of real wealth such as $\bar{\lambda} > \lambda'$ both industry 1 and 2 are able to offer the same marginal net job benefits in which case workers choose hours allocation across industries according to Proposition 8. Finally for even higher marginal values of real wealth such as $\lambda'' > \bar{\lambda}$, industry 2 offers the highest marginal net job benefits, in which case industry 1 is unable to operate. □

We now turn to the determination of labor-earnings supply and demand. LE^D is a simple extension of its earlier version. In particular, this function now satisfies

$$\lambda = U'\left(rM + \Pi + H\left(\xi W_1 + (1-\xi)\, W_2\right)\right). \tag{28}$$

Above, ξ is is the fraction of total work hours that a representative household devotes to industry 1. LE^S is slightly different than before. Recall from Figure 16 that for lower values of λ, such as λ', only industry 1 operates, and real wages and marginal net job benefits (hence, work hours as well) are relatively low. Thus, lower values of λ are associated with lower labor earnings, all else equal. Moreover, there is a critical (higher) λ at which both industries operate and both real wages and marginal net job benefits are higher, implying a perfectly elastic portion in labor-earnings supply. Finally, for even higher values of λ, such as λ'', only industry 2 is operational with associated higher wages and marginal net job benefits. This means that relatively high values of λ are associated with high values of labor earnings.

Figure 17 shows a configuration of LE^D and LE^S under which both industries operate. Within this setting, the implications of two comparative statics are particularly interesting: changes in relative prices, and changes in exogenous wealth. An increase in the relative price of good 1, say, through a simultaneous increase in p_1 and decrease in p_2, results in a horizontal expansion of industry 1's job utility function and a horizontal contraction in industry 2's job utility function. It immediately follows that the lower portion of LE^S shifts out, the upward portion of LE^S shifts back. Therefore, the elastic portion of LE^S - that is, the range over which the worker is indifferent between industries - shrinks. Suppose instead that non-interest, non-labor income Π increases. LE^S is not affected. However, LE^D shifts back. This means that every single bit of the increase in Π is devoted towards shifting hours of work to the industry offering a lower wage but higher job utility. In fact, if the magnitude of the shift in LE^D is sufficiently large, it could well be the case that in the new equilibrium the industry that used to offer lower job utility is no longer in operation.

5.3 Firm-Level Incentives for Drudgery Declines

At this stage, a natural question is whether incentives are in place for the development of innovations that lead to decreases in drudgery. To explore this issue, continue to assume the existence of a representative household. However, suppose that there is a continuum of firms indexed by i with production function $Y_i = K_i^{\alpha} \left(Z_i E_i H_i N_i\right)^{1-\alpha}$. Let the firms under consideration be monopolistic competitors producing intermediate inputs that are used in the production of a final good Y. In particular, assume $Y = \left(\int Y_i^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)} di$, where $\varepsilon > 1$, and there are no other factors used in the production of final output. It is straightforward to show that the optimal demand for input i satisfies $Y_i = Y/p_i^{\varepsilon}$, where p_i is the input's price.

Given monopolistic competition and the effective cost function in equation (10), profit maximization at the intermediate inputs stage solves, for any firm i,

$$\max_{Y_i} \left(p_i\left(Y_i\right) - \left(R^{\alpha}/\left(\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\right)\right)\omega_i^{1-\alpha}\right)\cdot Y_i. \tag{29}$$

Using $Y_i = Y/p_i^{\varepsilon}$ this problem's first-order condition implies that for each firm

$$p_i = \left(\varepsilon/\left(\varepsilon-1\right)\right)\left(R^{\alpha}/\left(\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\right)\right)\omega_i^{1-\alpha}. \tag{30}$$

Therefore, once the firm's effective wage is established, so is the price of its output and its level of production. Clearly, firms with the lowest effective wage will have the lowest price for their output, and hence a higher demand for their production.

As before, assume firms take as given the equilibrium level of hourly net job benefits B and the economy's marginal value of real wealth λ. A firm's optimization subproblem is to choose W_i and E_i to minimize $\omega_i = W_i/\left(PZ_iE_i\right)$ such that $\lambda W_i + J_i\left(E_i, D_i\right) = B$. Firm-level optimality is captured by $J_{iE} = -\lambda PZ_i\omega_i$. Given equation (30), under imperfect competition effective wages can indeed vary across firms. Since there is an economy-wide B, it follows that workers will decide how to allocate their time across firms with a decision rule analogous to that shown earlier.

Under imperfect competition changes in labor-augmenting technology and drudgery that affect one firm need not affect all or any other firms. This may be the result of these variables being protected by individual firms, for example, by secrecy or through patent laws. Therefore, in what follows, we focus on firm-specific comparative statics.

Proposition 10. *For any sign of J_{iED}, under imperfect competition the marginal value of real wealth held fixed effect of a decrease in drudgery is to decrease the effective wage.*

Proof. The firm's choice set expands. □

It follows that the overall result of a decrease in drudgery is that the firm expands: the decrease that occurs in the effective wage induces a decrease in the firm's marginal cost and therefore a decrease in the price of its output. Hence, Proposition 10 implies that firms with lower drudgery (or higher job utility per unit of effort) have a competitive advantage. To the extent that decreases in drudgery further this competitive advantage, it is even plausible that firms might set above-optimal effort requirements in order to induce workers themselves to think of ways to decrease drudgery. This amounts to a costless form of research and development.

As shown in the appendix, conditional on the sign of J_{iED} several different results can emerge given a change in drudgery. We limit ourselves to noting the interesting case shown in Figure 18, which depicts the λ held constant effects of a decrease in drudgery when $J_{iED} < 0$ and $J_{iE}dE_i/dD_i > -J_{iD}$. The latter condition amounts by total differentiation of the job utility function to $dJ_i/dD_i > 0$ and, as the appendix shows in detail, to $dW_i/dD_i < 0$. The case under consideration is such that decreases in drudgery make marginal effort less taxing on job utility. Under these circumstances a decrease in drudgery results in an increase in effort requirements, a decrease in job utility and the effective wage, and an increase in the real wage. These results are particularly interesting if one were to consider two firms, say 1 and 2, for which $D_1 > D_2$. Then, firm 2 would demand higher effort, offer a lower level of job utility, and pay a higher real wage. However, note that firm 2's jobs would actually offer higher job utility at any *given* effort level. Moreover, a comparison of firms would show that lower drudgery is associated with higher real wages. This highlights issues related to workers' comparison of jobs in terms of pleasantness. If individuals think of more pleasantness as lower drudgery, then as shown above they may report that more pleasant jobs also offer higher wages. This is also true if workers think of pleasantness as job utility per effort.

6 The Role of Amenities

For ease of exposition we revert to the context of a representative household and final-good producer, where the firm is perfectly competitive in the product market. Recall that we have defined amenities to be job characteristics whose costs are in terms of goods. Denote by p_A the price of amenities relative to the final consumption good, and by A the level of amenities per hour of work that the firm offers to each employee. Let $J(E, D, A)$ be the job-utility function extended to account for amenities, and assume $J_A > 0$, $J_{AA} < 0$, and the same properties over E and D as $J(E, D)$.

Following steps analogous to those taken earlier in the paper, the solution to the worker's labor-hours supply optimization subproblem is just as before. In turn, the firm's cost minimization problem is given by

$$\min_{K,\, HN} RK + \mathcal{W}HN \tag{31}$$

such that $K^{\alpha}(ZEHN)^{1-\alpha} = \bar{Y}$, where $\mathcal{W} = W + p_A A$ is the *inclusive wage*. The relevant cost function becomes

$$\mathcal{C}\left(\omega, R, \bar{Y}\right) = \left(R^{\alpha}/\left(\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right)\right)\omega^{1-\alpha}\bar{Y}, \tag{32}$$

which is similar to the one derived in Section 3 except that now $\omega = \mathcal{W}/(ZE)$ is the appropriate version of the effective wage.

The firm's new optimization subproblem is to choose a real wage W, effort per worker E, and amenities per worker A to minimize $\omega = \mathcal{W}/(ZE)$ subject to $\lambda W + J(E, A, D) = B$. Let ψ be the multiplier associated with the firm's optimization subproblem. Then, the first-order conditions are

$$W : 1/ZE - \psi\lambda = 0,\ A : p_A/ZE - \psi J_A = 0, \text{ and } E : -(W + p_A A)/ZE^2 - \psi J_E = 0. \tag{33}$$

Combine the first and last first-order conditions to yield $EJ_E = -\lambda(W + p_A A)$. Dividing this by E, and multiplying and dividing the right side by Z yields, in (E, J) space, the exact same optimality condition as earlier in the paper: $J_E = -\lambda Z\omega$. In addition, combining the first and second of the first-order conditions implies that $p_A\lambda = J_A$. Together, these last two equations implicitly define the firm's optimal choice of effort, amenities, and real wage given the exogenous parameters λ, Z, p_A, and D. Alternatively, the optimality conditions $J_E = -\lambda Z\omega$ and $p_A\lambda = J_A$ can be combined to eliminate λ and yield the expression $-\varepsilon_{JE} = \varepsilon_{JA}\left(1 + W/p_A A\right)$, where for any variables x and y, $\varepsilon_{xy} = d\log x/d\log y$. Hence, at the firm's optimal choices the absolute value of the elasticity of job utility with respect to effort, ε_{JE}, equals that with respect to amenities, ε_{JA}, weighted by 1 plus the ratio of the hourly per worker real wage to the hourly cost of amenities per worker.[14]

The abstract functional form $J\left(E, D, A\right)$ is not as agreeable for graphical analysis as was the case without amenities. Consider, however, a special case that proves illuminating. Suppose that $J\left(E, D, A\right) = G\left(E, D\right) + F\left(A\right)$. The first-order condition for amenities and the real wage together imply that $p_A\lambda = J_A$, which in this case amounts to $p_A\lambda = F'\left(A\right)$. Moreover, note that

$$\lambda W = \lambda\left(\mathcal{W} - p_A A\right) = \lambda\left(\omega Z E - p_A A\right). \tag{34}$$

Therefore,

$$\lambda\omega Z E + G(E, D) + F\left(A\right) - \lambda p_A A = B, \tag{35}$$

and the firm's optimal choice of amenities is alternatively the result of an optimization problem in which A is chosen to maximize $F\left(A\right) - \lambda p_A A$. Let

$$S\left(\lambda p_A\right) = \max_A\left\{F\left(A\right) - \lambda p_A\right\} = F\left(F'^{-1}\left(\lambda p_A\right)\right) - \lambda p_A F'^{-1}\left(\lambda p_A\right) \tag{36}$$

be the surplus an individual receives from the optimal choice of amenities, and note that $S_{p_A}, S_\lambda < 0$. In $(E, G+S)$ space the firm's isocost lines now satisfy $G + S = B - \lambda Z\omega E$. As before the less steep this line, the lower the associated effective wage. Moreover, the optimality condition for effort, $J_E = -\lambda Z\omega$, is such that in $(E, G+S)$ space the slope of the job utility function $J_E = G_E$ is equal to the slope of an isocost line $-\lambda Z\omega$. In other words, once the level of amenities is determined, the solution the firm's problem in the present

[14] It is straightforward to show that the optimal level of amenities is the same if instead households are assumed to choose A.

context is entirely analogous to that which we presented earlier. This is shown in Figure 19.[15]

Given the direct relationship between amenities and λ, we examine the differences between an economy with marginal value of real wealth λ and one with $\lambda' < \lambda$. Of course, a lower marginal value of real wealth is consistent with higher amenities, meaning that under λ' the firm's job utility function in $(E, G + S)$ space shifts up and its isocost lines become less steep. This is shown in Figure 20, where lower effort and higher job utility under λ' are also noted. Since E is lower and both Z and E remain constant, it follows that under λ' the real wage is lower. Although it may seem ambiguous, as shown in the appendix marginal net job benefits are actually lower under λ', meaning that so are equilibrium work hours. Note however, that given the endogenously optimal higher A consistent with a lower λ, the difference in equilibrium work hours between steady states is less than in the absence of amenities. In that sense, amenities can be seen as partially muting declines larger declines in work hours that would otherwise occur given decreases in the marginal value of real wealth.[16]

7 Work Hours and Welfare

7.1 Equilibrium Work Hours

Recall that work hours are a direct function of marginal net job benefits B. Assuming a continuum of firms indexed by i, and using the ongoing definition of B, it follows that

$$dB = (\lambda dW_i + dJ_i) + W_i d\lambda. \tag{37}$$

Above, the first two terms imply that both increases in real wages and on-the-job utility will induce individuals to work more hours. However, the third term shows that increases

[15]It is straightforward to rederive all comparative statics as developed earlier in the paper once amenities are accounted for.

[16]For simplicity, we have not considered the production-side of amenities. However, note that if these are interpreted as fractions of the consumption good transformed into amenities, then $p_A = 1$. Otherwise, for example, the sectoral analysis developed earlier can be applied in straightforward fashion. Whichever the case, the main points of this section are not altered.

in consumption - and therefore decreases in the marginal value of real wealth λ - do the opposite.

It follows that within this framework, the extent to which work hours remain high, and for that matter higher than expected given enormous secular increases in consumption, is a reflection of ongoing increases in $\lambda W_i + J_i$. Changes in this sum can be operationalized since dB captures changes in hours per worker, $d\lambda$ changes in consumption, and $dB - W_i d\lambda$ equals $\lambda dW_i + dJ_i$.

Labor labor hours are consistent with $dB = 0$, which implies that $dJ_i = -W_i d\lambda - \lambda dW_i$. If income effects dominate substitution effects, then $W_i d\lambda < -\lambda dW_i$ holds. If this is true, then $dB = 0$ if and only if $dJ_i > 0$. That is, if and only if job utility is increasing. In addition, note that even if $\lambda W_i \rightarrow 0$ because the income effect overwhelms the substitution effect, work hours H_i will tend to some constant $\bar{H}_i > 0$ as long as job utility J_i tends to some constant $\bar{J}_i > \Phi'(0)$. That is, the model can in principle explain a positive asymptote for work hours if people enjoy work as much as the marginal leisure activity.

The data suggests relatively trendless labor hours in the face of large secular increases in consumption and real wages. As argued, for instance, in Kimball and Shapiro (2008), income effects on labor supply are indeed substantial. This means, in particular, that if the income effect dominates the substitution effect, labor hours will be relatively trendless if and only if there are ongoing upward shifts in firms' job-utility functions. We have shown that such outward shifts can be triggered by decreases in drudgery or increases in amenities. Moreover, there are strong firm-level microeconomic incentives to focus on innovations that decrease drudgery, and amenities are inversely related to the economy's marginal value of real wealth. This means that as economies become richer, amenities are expected to increase, the direct effect of which is to partially mute income effects that would otherwise lead to large decreases in work hours. Since both decreases in drudgery and increases in amenities shift the job-utility function outwards, our analysis suggests intuitive channels through which observed patterns in the data can be explained.

7.2 Welfare Under Additive Separability

In order to address the theory's welfare implications we allow for a variety of job options so that $H = \sum_i H_i$. Parameter-induced changes in welfare are well assessed via steady-state to steady-state considerations. In steady state, given $r = \rho$, an individual's problem is equivalent to the static optimization problem

$$\max_{C,\ H,\ H_i \geq 0} U + \Phi + \textstyle\sum_i H_i J_i \tag{38}$$

such that

$$C = rM + \Pi + \textstyle\sum_i W_i H_i \tag{39}$$

and total hours

$$H = \textstyle\sum_i H_i. \tag{40}$$

Given the multipliers λ and b, let

$$\mathcal{L}^* = \max_{C,\ H,\ H_i \geq 0} \{U + \Phi + \textstyle\sum_i H_i J_i + b\left(H - \sum_i H_i\right) + \lambda\left(rM + \Pi + \sum_i W_i H_i - C\right)\}.$$

Note that the optimal choice of H_i yields two cases: $H_i = 0$ and $J_i + \lambda W_i < b$, or $H_i > 0$ and $J_i + \lambda W_i = b$. Therefore, $b = B$, where, as before, B denotes the economy's level of equilibrium marginal net job benefits.

Using the envelope theorem,

$$d\mathcal{L}^*/\lambda = \textstyle\sum_i H_i dJ_i/\lambda + \sum_i H_i dW_i + d\left(\Pi + rM\right) \tag{41}$$

Above, each of the three terms on the right-hand-side highlights a distinct way in which the economy's opportunity set becomes larger. Changes in welfare owing to changes in on-the-job utility are captured by the first term, modifications due to increases in consumption are reflected in the second term, and modifications owing to changes in exogenous wealth appear in the last term. In particular, $\left(\sum_i H_i\right) dJ_i/\lambda$ can be interpreted as the portion of the change in the maximized value of utility that answers the question of how much the household has to be paid in order to go back to working in yesterday's conditions.

To better understand the implications of the envelope theorem, note that

$$d\left(\sum_i H_i W_i\right) = \sum_i H_i dW_i + \sum_i W_i H_i \cdot dH/H + \sum_i W_i \left(dH_i - H_i \cdot dH/H\right) \tag{42}$$

That is, the change in labor earnings is equal to the sum of a term reflecting the change in wages for narrowly defined job categories, a term reflecting the change in total hours, and a term reflecting the change in the composition of jobs between relatively high paid jobs with low job utility and low paid jobs with high job utility. The change in wages for narrowly defined jobs is a key component of welfare from the envelope theorem perspective. Note that

$$\sum_i H_i dW_i = d\left(\sum_i H_i W_i\right) - \sum_i W_i H_i \cdot dH/H - \sum_i W_i \left(dH_i - H_i \cdot dH/H\right). \tag{43}$$

Therefore, to gauge this component of welfare, we need to adjust the change in overall labor earnings by subtracting not only extra earnings from people working longer hours overall, but also extra earnings coming from people switching towards jobs that are more highly paid and have lower job utility. If λW is moving down, then the overall trend should involve compositional shifts towards jobs with higher job utility and relatively lower pay than other available jobs. This means that the increase in labor earnings will tend to understate the true increase in welfare (leaving aside changes in overall hours). Empirically, it should be possible to obtain a direct measure of the change in wages for narrowly defined jobs $\sum_i H_i dW_i$.

In terms of the remaining welfare components, consider once more equation (37). Using this, rearranging, and substituting in equation (41) implies that

$$\begin{gathered} d\mathcal{L}^*/\lambda = \left(\sum_i H_i\right) dB/\lambda - \left(\sum_i H_i W_i\right) d\lambda/\lambda + d\left(\Pi + rM\right) \\ \Longrightarrow \frac{d\mathcal{L}^*}{\lambda \sum_i H_i W_i} = \frac{H}{\sum_i H_i W_i} \frac{dB}{\lambda} - \frac{d\lambda}{\lambda} + \frac{d\left(\Pi + rM\right)}{\sum_i H_i W_i}. \end{gathered} \tag{44}$$

The last term on the right-hand side is well understood. As noted above, $\sum_i H_i dW_i$ can in principle be computed. Hence, we would like a measure for the first two terms on the right-hand side of equation (44).

Define $\gamma = -CU_{CC}/U_C$. Then, $1/\gamma$ is the elasticity of intertemporal substitution, and $d\lambda/\lambda = -\gamma dC/C$. Moreover, for any job i the Frisch elasticity of labor supply satisfies $\eta_i (1-\zeta_i)$. Then,

$$dB/B = (1/\bar{\eta})\, dH/H \implies dB = ((1-\zeta_i)\, \lambda W_i)\, dH/H \implies dB/\lambda = (W_i/\eta_i)\, dH/H. \quad (45)$$

Substituting these derivations into equation (44) and simplifying yields

$$\frac{d\mathcal{L}^*}{\lambda \sum_i H_i W_i} = \frac{(W_i/\eta_i)\, dH}{\sum_i H_i W_i} + \frac{\gamma dC}{C} + \frac{d\,(\Pi + rM)}{\sum_i H_i W_i}. \quad (46)$$

Evidence about γ can be found from workers' job choices. Consider an individual working two jobs satisfying $J_2 > J_1$. Then, $\lambda W_1 + J_1 = \lambda W_2 + J_2$, meaning that

$$\lambda = \frac{J_2 - J_1}{W_2 - W_1} \implies \frac{d\lambda}{\lambda} = \frac{dJ_1 - dJ_2}{J_1 - J_2} - \frac{dW_1 - dW_2}{W_1 - W_2}. \quad (47)$$

For any individual with $dJ_1 - dJ_2 = 0$, for example, $dJ_1, dJ_2 = 0$, then

$$d\lambda/\lambda = -\,(dW_1 - dW_2)\,/\,(W_1 - W_2)\,, \quad (48)$$

and using $d\lambda/\lambda = -\gamma dC/C$ it follows that

$$\gamma = ((dW_1 - dW_2)\,/\,(W_1 - W_2))\,/\,(dC/C)\,. \quad (49)$$

The short-run elasticity of intertemporal substitution has been suggested by Hall (1988) to be approximately zero, and by Kimball, Sahm, and Shapiro (2011) to be 0.08. However, there are reasons suggesting that the long-run elasticity of intertemporal substitution should be higher than its short-run counterpart. This includes taking account of full adjustment, new goods, habit formation, and "keeping up with the Joneses." In the context of our analysis, it is precisely the long-run elasticity of intertemporal substitution which should be applied. Say the long-run elasticity of intertemporal substitution is 0.5, in which case $\gamma = 2$. Using this value for γ along with equation (46) implies that for $d\Pi, dM = 0$ and $dH = 0$, a

1% increase in consumption would be associated with a welfare increase of at least 2%.

A natural question that follows is what fraction of welfare gains are attributable to higher job utility. To see this, note that dividing equation (41) by $\sum_i H_i W_i$ and combining with equation (46) yields

$$\begin{aligned} \frac{\sum_i H_i dJ_i}{\lambda \sum_i H_i W_i} + \frac{\sum_i H_i dW_i}{\sum_i H_i W_i} &= \frac{(W_i/\eta_i)\, dH}{\sum_i H_i W_i} + \frac{\gamma dC}{C} \\ \Longrightarrow \frac{\sum_i H_i dJ_i}{\lambda \sum_i H_i W_i} + \left(\frac{d \sum_i H_i W_i}{\sum_i H_i W_i} - \frac{\sum_i W_i dH_i}{\sum_i H_i W_i} \right) &= \frac{(W_i/\eta_i)\, dH}{\sum_i H_i W_i} + \frac{\gamma dC}{C}. \end{aligned} \tag{50}$$

Then, continuing to assume $\gamma = 2$ and given $dH = dH_i = 0$, a 1% increase in consumption resulting from a 1% increase in labor earnings (that is, with *all* of the increase in labor earnings being put towards consumption) implies that

$$\frac{\sum_i H_i dJ_i}{\lambda \sum_i H_i W_i} = \frac{\gamma dC}{C} - \frac{d \sum_i H_i W_i}{\sum_i H_i W_i} = 2\% - 1\% = 1\%. \tag{51}$$

Hence, given constant labor hours, in the present example up to half of the welfare gains associated with a 1% increase in consumption can result from increases in on-the-job utility.

7.3 Welfare Under Non-separability

Finally, it is of interest to understand the welfare implications of job utility when consumption and leisure are non-separable. Let

$$\mathcal{U} = \mathbb{U}\,(C, H) + \textstyle\sum_i H_i J_i, \tag{52}$$

where $\mathbb{U}_C > 0$, $\mathbb{U}_H < 0$, and $\mathbb{U}_{CH} > 0$. Then, an individual's problem involves choosing C, H, and $H_i \geq 0$ to maximize $\mathcal{U}$ subject, once again, to the constraints in equations (39) and (40). Let

$$\mathcal{L}^* = \max_{C,\, H,\, H_i} \; \mathbb{U}\,(C, H) + \textstyle\sum_i H_i J_i + \lambda \left(rM + \Pi + \sum_i W_i H_i - C \right) + B \left(H - \sum_i H_i \right). \tag{53}$$

Then,

$$d\mathcal{L}^* = \sum_i H_i \left(dJ_i + \lambda dW_i\right) + \lambda \left(d\Pi + r dM\right). \tag{54}$$

Using equation (37), summing over hours, and dividing by λC the previous can be stated as

$$d\mathcal{L}^*/\lambda C = (H/C) \cdot dB/\lambda - \left(\sum_i H_i W_i/C\right) \cdot d\lambda/\lambda + \left(d\Pi + r dM\right)/C. \tag{55}$$

Other than dB/λ and $d\lambda/\lambda$, it is straightforward to obtain empirical counterparts to all variables on the right-hand side of the equation (55). It is therefore of interest to find expressions for dB/λ and $d\lambda/\lambda$ that can be operationalized. To this end, define

$$V(\lambda, H) = \max_C \mathbb{U}(C, H) - \lambda C \tag{56}$$

and consider the expression

$$\max_H V(\lambda, H) + \lambda \left(H \sum_i \xi_i W_i + \Pi\right) + H \sum_i \xi_i J_i, \tag{57}$$

where ξ_i is the fraction of total hours that the individual spends on job i. Note that

$$H\left(\lambda \sum_i \xi_i W_i + \sum_i \xi_i J_i\right) = H \sum_i \xi_i \left(\lambda W_i + J_i\right) = HB \tag{58}$$

since in equilibrium $B = \lambda W_i + J_i$, and also $\sum_i \xi_i = 1$. Therefore, the statement in equation (57) becomes

$$\max_H V(\lambda, H) + H\left(\lambda \sum_i \xi_i W_i + \sum_i \xi_i J_i\right) + \lambda \Pi. \tag{59}$$

The first-order condition is $-V_H(\lambda, H) = B$. Hence, $dB = -V_{HH} dH - V_{H\lambda} d\lambda$. If $d\lambda = 0$, then

$$dB/B = -\left(V_{HH} H/B\right) \cdot dH/H = \left(V_{HH} H/V_H\right) \cdot dH/H, \tag{60}$$

where the second equality follows from the earlier first-order condition. It follows that,

$$(dB/B)/(dH/H) = \left(V_{HH} H/V_H\right) = 1/\bar{\eta}, \tag{61}$$

where $\bar{\eta}$ is defined as the λ-held-constant elasticity of H with respect to B. Given

$$dB = -V_{HH}dH - V_{H\lambda}d\lambda, \tag{62}$$

as shown in the appendix

$$dB/\lambda = (1-\zeta_i)\, W_i/\bar{\eta} \cdot dH/H - V_{H\lambda} \cdot d\lambda/\lambda, \tag{63}$$

where $\zeta_i = -J_i/W_i\lambda$.

From Proposition 1, the Frisch elasticity of labor supply for any job i is given by $\eta_i = \bar{\eta}/\left(1-\zeta_i\right)$. Therefore the previous can be stated as,

$$dB/\lambda = (W_i/\eta_i) \cdot dH/H - V_{H\lambda} \cdot d\lambda/\lambda. \tag{64}$$

The first term on the right-hand side above has straightforward empirical counterparts. However, we still require an expression for $d\lambda/\lambda$, and are now also in need of one for $V_{H\lambda}$. Note from the expression in (56) that $V_\lambda = -C\left(\lambda, H\right)$ and $V_{\lambda H} = -C_H\left(\lambda, H\right)$. Furthermore, as shown in in the appendix

$$dC/C = (V_{\lambda\lambda}\lambda/V_\lambda) \cdot d\lambda/\lambda + (V_{\lambda H}H/V_\lambda) \cdot dH/H. \tag{65}$$

Define $-1/\gamma = V_{\lambda\lambda}\lambda/V_\lambda$ and $\Theta = V_{\lambda H}H/V_\lambda$. That is, $\Theta = d\ln C/dH$ for constant λ. Then,

$$d\lambda/\lambda = \gamma\left(\Theta \cdot dH/H - dC/C\right). \tag{66}$$

A value for Θ can be estimated by noting that

$$\Delta \ln C + \alpha + \beta r + \Theta\Delta \ln H + \varepsilon. \tag{67}$$

Basu and Kimball (2002) suggest that a higher-end estimate for Θ is 0.3. Moreover,

$$\Theta = V_{\lambda H} H / V_{\lambda} = -V_{\lambda H} H / C \implies -V_{\lambda H} = \Theta C / H \tag{68}$$

Substituting into equation (64),

$$\begin{aligned} dB/\lambda &= (W_i/\eta_i) \cdot dH/H - V_{H\lambda} \cdot d\lambda/\lambda \\ &\implies dB/\lambda = (W_i/\eta_i) \cdot dH/H + (\Theta C/H) \cdot d\lambda/\lambda. \end{aligned} \tag{69}$$

We set out to search for empirical counterparts to $d\lambda/\lambda$ and dB/λ for use in equation (55), which we now have in equations (66) and (69). Combining these three equations, as shown in the appendix it now follows that

$$\frac{d\mathcal{L}^*}{\lambda C} = \frac{W_i}{\eta_i}\frac{dH}{C} + \left(\Theta - \frac{\sum_i H_i W_i}{C}\right)\gamma\left(\Theta \cdot \frac{dH}{H} - \frac{dC}{C}\right) + \frac{(d\Pi + r dM)}{C}. \tag{70}$$

Consider an example. Suppose $dC/C = 1\%$, $dH = 0$, $\Theta = 0.3$, $\gamma = 2$. Moreover, suppose $\sum_i H_i W_i = C$ so that there is no non-labor income, and $d\Pi = dM = 0$. Then, using equation (70)

$$d\mathcal{L}^*/\lambda C = (.3 - 1) \cdot 2 \cdot (-1\%) = 1.4\%. \tag{71}$$

Hence, in this case .4 percentage points beyond the welfare increase stemming from the increase in consumption owes to changes in on-the-job utility. Note that in terms of welfare there is no fundamental difference between increases in J from compositional effects and increases in J in any given job - it is only a matter of how detailed the definitions of jobs are.

8 Conclusions

The *paradox of hard work* refers to the fact that, given enormous world-wide increases in consumption, work hours have remained relatively trendless across countries. Given a low elasticity of intertemporal substitution[17] and income effects on labor supply being substantial,[18] work hours should have exhibited a substantial decline. In principle, the lack of such decline can rationalized by assuming that the elasticity of intertemporal substitution is large, by an increasing marginal-wage to consumption ratio, by something that keeps the marginal utility of consumption high, or by something that keeps the marginal disutility of work low. We focus attention on the last of these explanations. Economists have long understood that cross-sectional differences in on-the-job utility at a particular time give rise to compensating differentials. In this paper, we develop a theory that focuses on a less-studied topic: understanding the long-run macroeconomic consequences of trends in on-the-job utility. Two main implications emerge. First, secular improvements in on-the-job utility are such that work hours can remain approximately constant over time even if the income effect of higher wages on labor supply exceeds the substitution effect of higher wages. Second, secular improvements in on-the-job utility can themselves be a substantial component of the welfare gains from technological progress. These two implications are connected by an identity: improvements in on-the-job utility that have a significant effect on labor supply tend to have large welfare effects.

The major analytical developments in this paper are based on a model that allows us to study the interaction of *work hours* (which stands in for all aspects of the job that interfere with leisure and home production), *effort* (which stands in for all aspects of a job whose cost is in terms of proportionate changes in effective productive input from labor), *amenities* (which we define to be job characteristics whose cost is in terms of goods), and *drudgery* (which is a variable capturing everything else that matters for job utility). Once job utility is explicitly accounted for, the economy's general equilibrium follows by way of two novel theoretical objects: *labor-earnings supply and labor-earnings demand.*

This paper's research contributes to the labor economics literature by developing a theoretical framework through which an intertemporal understanding of the primitives that determine the economy's available trade-offs between output, wages, and job utility can be attained. Moreover, we contribute to the macroeconomics literature by offering a novel ex-

[17] See, for example, Hall (1988), Barsky et al. (1997), and Basu and Kimball (2002).
[18] See, for example, Kimball and Shapiro (2008).

planation for the paradox of hard work, thus showing that this paradox is not necessarily evidence of a large intertemporal elasticity of substitution or non-separable preferences in consumption and leisure.

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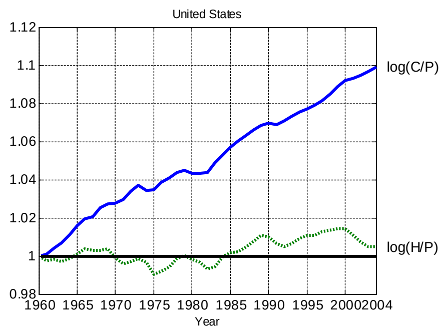
A Figures



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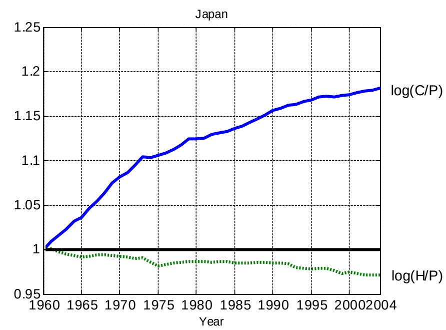

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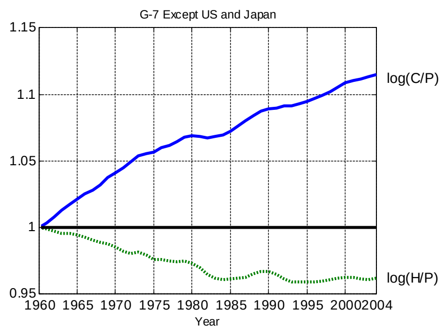

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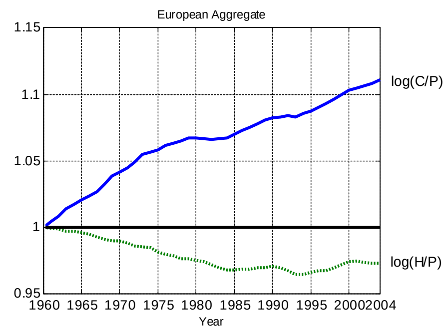

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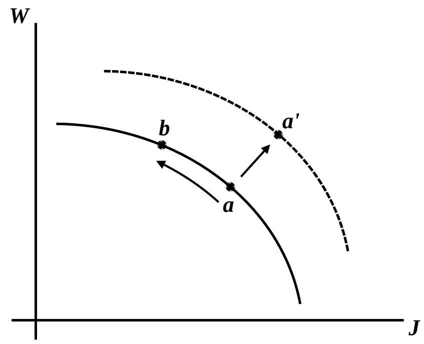

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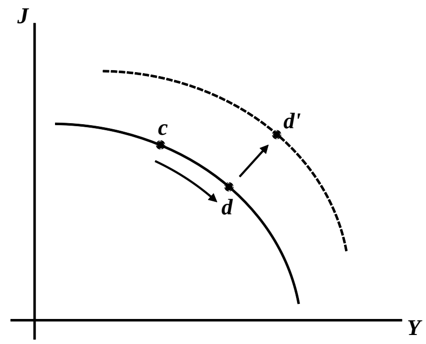

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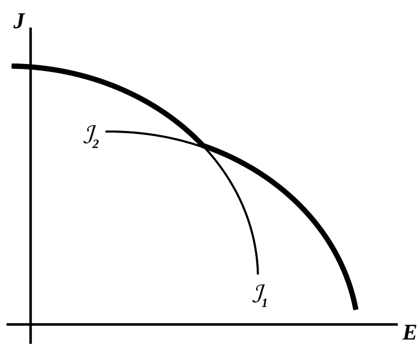

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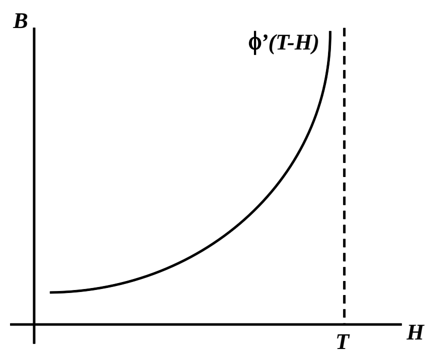

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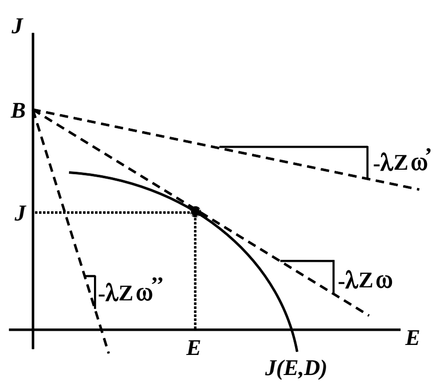

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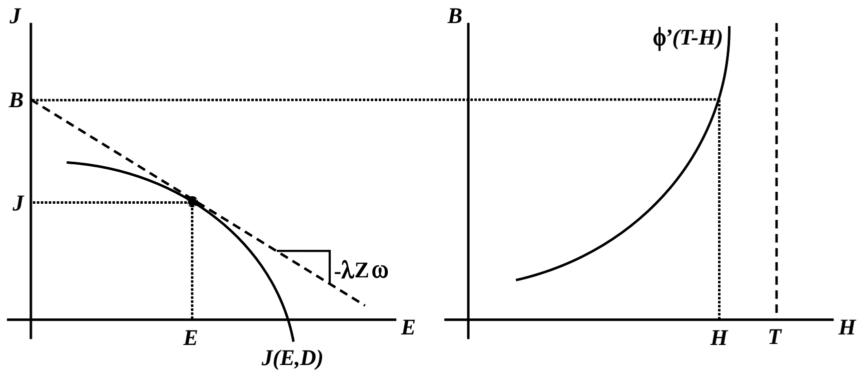

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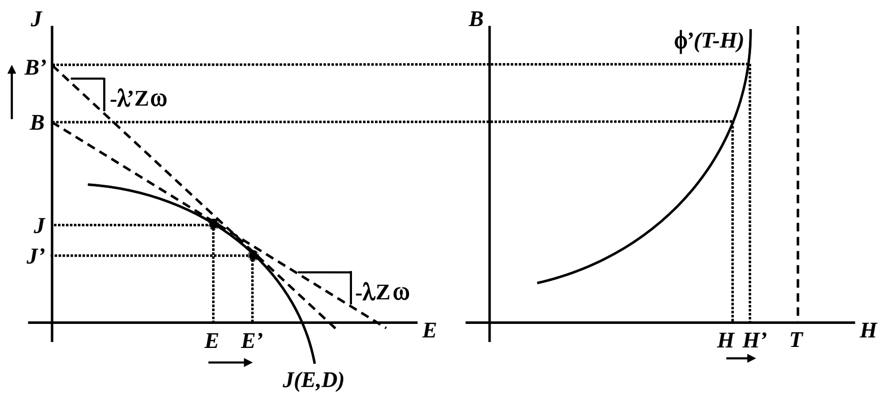

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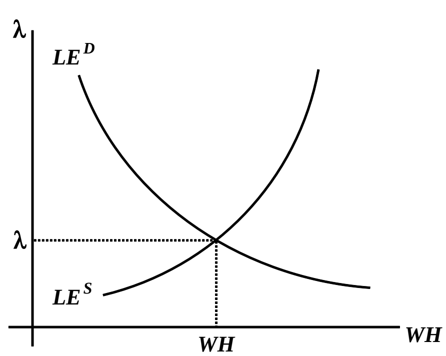

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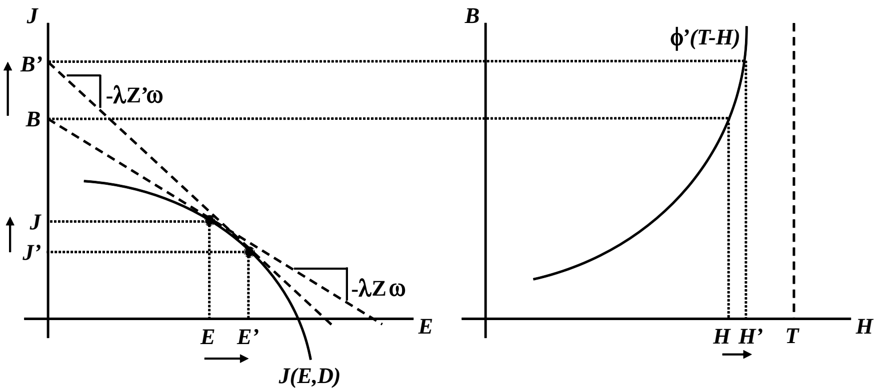

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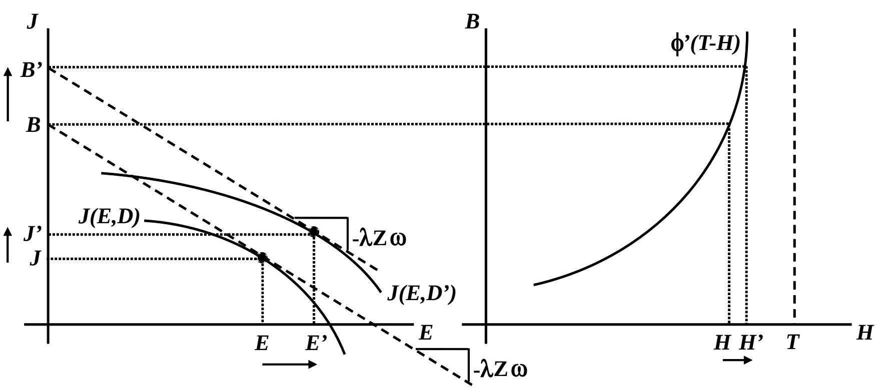

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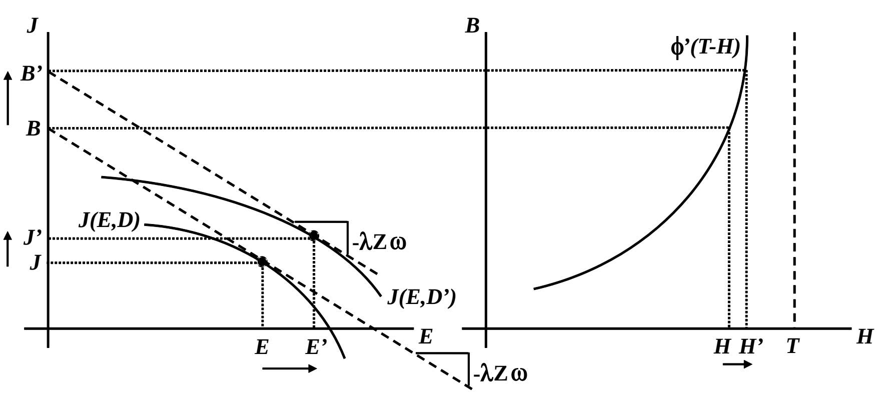

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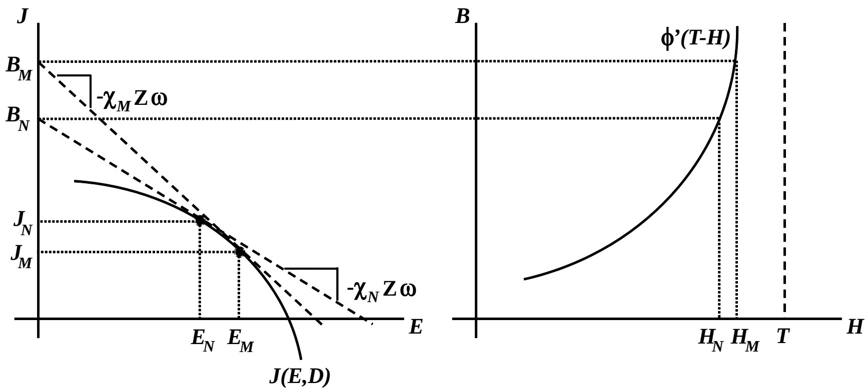

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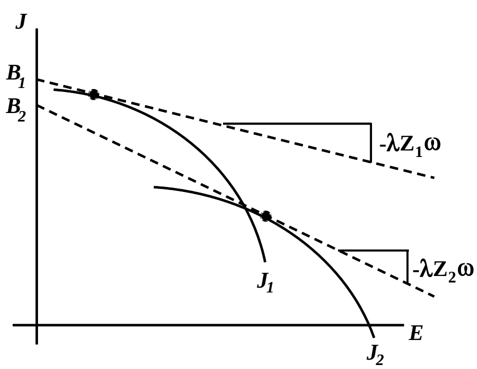

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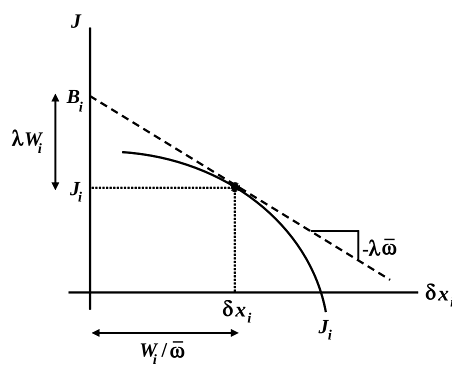

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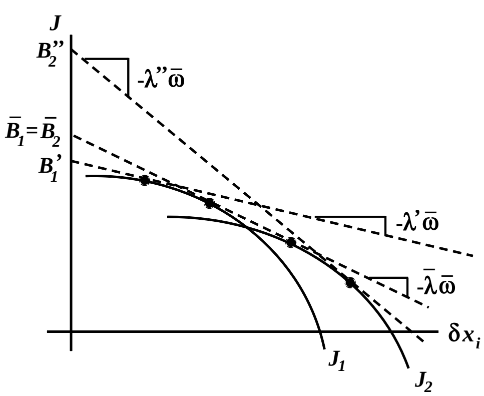

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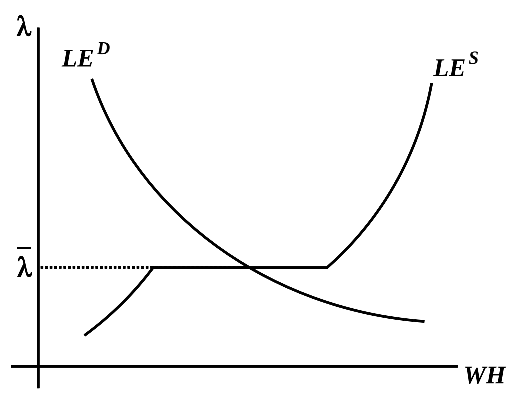

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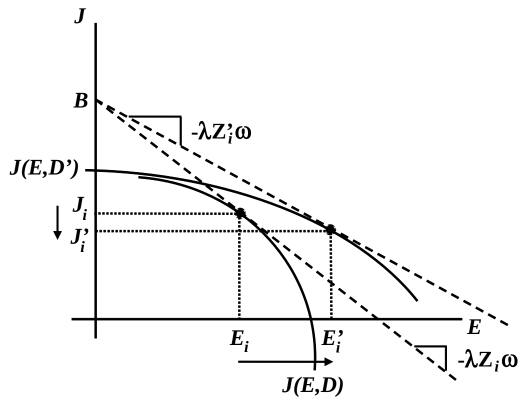

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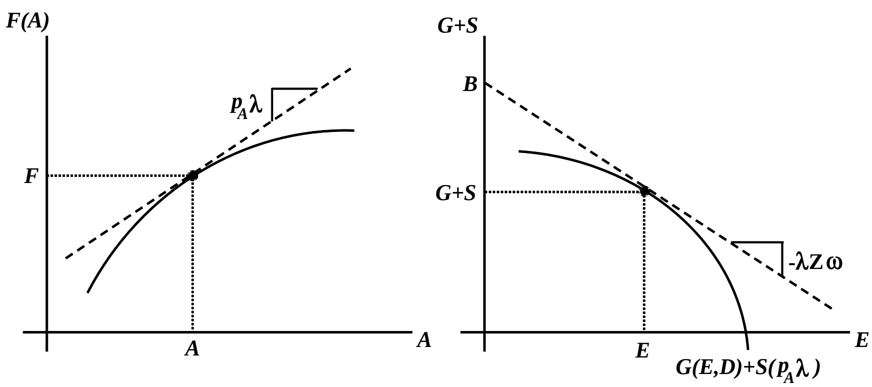

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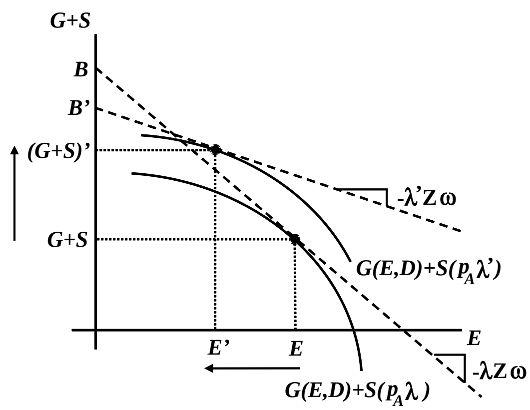

Figure 20

B Derivations

B.1 Normalization

Consider $U + \tilde{\Phi} + H\tilde{J}$ with $\tilde{\Phi}'(T) = \kappa$, where κ is a constant. Define $\Phi(X) = \tilde{\Phi}(X) - \kappa X$ and $J = \tilde{J} - \kappa H$. Then, $\Phi'(T) = 0$, and

$$\mathcal{U} = U + \tilde{\Phi}(T - H) + H\tilde{J} - \kappa(T - H) - \kappa H \Longrightarrow \mathcal{U} = U + \tilde{\Phi}(T - H) + H\tilde{J} - \kappa T,$$

which is equivalent to $U + \tilde{\Phi} + H\tilde{J}$.

B.2 Proof of Proposition 8

$J_2 > J_1$ implies that $W_1 > W_2$. In equilibrium both firms offer the same B. Hence, $B = \lambda W_1 + J_1$ and $B = \lambda W_2 + J_2$. Combining yields

$$\lambda = -(J_1 - J_2)/(W_1 - W_2). \tag{72}$$

Using the household's budget constraint

$$C = H(\xi W_1 + (1-\xi) W_2) + rM + \Pi. \tag{73}$$

Since the household's choice of total work-hours supply satisfies $\Phi'(T-H) = B$, then $H = T - \Phi'^{-1}(B)$. Combining implies that

$$C = \left(T - \Phi'^{-1}(B)\right)(\xi W_1 + (1-\xi) W_2) + rM + \Pi. \tag{74}$$

Given the household's condition for optimal consumption, $U'^{-1}(\lambda) = C$. Combining these two final equations and rearranging yields the desired expression for ξ. □

B.3 Details on the Incentives for Drudgery Declines

Given a change in drudgery - and keeping all else constant, in particular equilibrium net job benefits -, total differentiation of the firm's constraint, $\lambda W_i + J_i = B$, yields

$$J_{iE} dE_i/dD_i + \lambda dW_i/dD_i = -J_{iD}. \tag{75}$$

Similarly, total differentiation of the optimality condition $-J_{iE}E_i = \lambda W_i$ implies that

$$J_{iE} dE_i/dD_i + \lambda dW_i/dD_i = -J_{iEE} E_i dE_i/dD_i - J_{iED} E_i. \tag{76}$$

Combining the previous two equations results in

$$dE_i/dD_i = (J_{iD} - J_{iED} E_i) / J_{iEE} E_i. \tag{77}$$

By assumption $J_{iD}, J_{iEE} < 0$. Whereas $J_{iED} \geq 0$ ensures that dE_i/dD_i is strictly positive, $J_{iED} < 0$ allows for $dE_i/dD_i \lesseqgtr 0$. Moreover, note that rearranging equation (75) implies that

$$dW_i/dD_i = -J_{iD}/\lambda - (J_{iE}/\lambda)\, dE_i/dD_i. \tag{78}$$

Before proceeding, totally differentiate the job function. This yields

$$dJ_i/dD_i = J_{iE}dE_i/dD_i + J_{iD}, \tag{79}$$

where we refer to $J_{iE}dE_i/dD_i$ as the *effort substitution effect* and J_{iD} as the *(pure) drudgery effect*. Note that whereas the drudgery effect is always negative, the sign of the effort substitution effect is ambiguous and depends directly on that of dE_i/dD_i.[19]

The effects of a change in drudgery depend on the sign of J_{iED}. Consider the case in which $J_{iED} < 0$. This initially gives way to three additional possibilities conditional on the sign of the numerator in equation (77). Assume $-J_{iD} < -J_{iED}E_i$. In this case, equation (77) implies that $dE_i/dD_i < 0$. However, by equation (75)

$$dW_i/dD_i = -J_{iD}/\lambda - (J_{iE}/\lambda)(dE_i/dD_i) \lesseqgtr 0.$$

Using equation (79) the fact that $dE_i/dD_i < 0$ implies that in this case the effort substitution and drudgery effects work in opposite directions. If the effort substitution effect dominates the drudgery effect, then $dJ_i/dD_i > 0$. Hence,

$$J_{iE}dE_i/dD_i > -J_{iD} \implies 0 > -(J_{iE}/\lambda)(dE_i/dD_i) - J_{iD}/\lambda$$

which using (78) implies that $dW_i/dD_i < 0$.

B.4 Amenities

To see that under λ' marginal net job benefits are lower than under λ, consider the firm's constraint $\lambda Z\omega E + G = B + F$. Since both ω and Z remain constant, it follows that

$$Z\omega E d\lambda + \lambda Z\omega dE + G_E dE = d(B - F + F'A).$$

[19] Recall that $J_{iE} < 0$ is an endogenous result of the firm's optimization subproblem given positive, as we have assumed throughout the essay.

The firm's optimality condition $G_E = -\lambda Z\omega$ implies that $G_E dE = -\lambda Z\omega d\lambda$. Using this in the differentiated version of the firm's constraint results, after rearranging, in $Z\omega E = dB/d\lambda + (F''A)\, dA/d\lambda$. Then, use of the optimality condition $p_A\lambda = F'$ implies that $dA/d\lambda = cp_A/F''$. Substituting this into the firm's differentiated constraint implies that the second term on its right side reduces to $p_A A$. Moreover, note that the left side of this equation is actually $(W + p_A A)\, ZE/ZE$. Given the previous, rearranging and simplifying the firm's differentiated constraint results in $dB/d\lambda = W > 0$ assuming, as we have throughout the paper, a positive real wage.

B.5 Welfare Under Non-Separability

Given

$$\frac{dB}{\lambda} = (-V_H/\lambda\bar{\eta}) \cdot dH/H - V_{H\lambda} \cdot d\lambda/\lambda,$$

it follows that

$$\begin{aligned} dB/\lambda &= (B/\lambda\bar{\eta}) \cdot dH/H - V_{H\lambda} \cdot d\lambda/\lambda \\ \implies dB/\lambda &= (\lambda W_i + J_i)/\lambda\bar{\eta} \cdot dH/H - V_{H\lambda} d\lambda/\lambda \\ \implies dB/\lambda &= (W_i + J_i/\lambda)/\bar{\eta} \cdot dH/H - V_{H\lambda} \cdot d\lambda/\lambda \\ \implies dB/\lambda &= (1 - \zeta_i)\, W_i/\bar{\eta} \cdot dH/H - V_{H\lambda} \cdot d\lambda/\lambda. \end{aligned}$$

Now, consider $C(\lambda, H) = -V_\lambda(\lambda, H)$. This implies that

$$dC = C_\lambda d\lambda + C_H dH$$

$$\implies dC = -V_{\lambda\lambda} d\lambda - V_{\lambda H} dH \implies dC/C = (V_{\lambda\lambda}\lambda/V_\lambda) \cdot d\lambda/\lambda + (V_{\lambda H} H/V_\lambda) \cdot dH/H.$$

Finally, note that

$$\frac{d\mathcal{L}^*}{\lambda C} = \frac{H}{C}\left(\frac{W_i}{\eta_i}\frac{dH}{H} + \left(\Theta\frac{C}{H}\right)\frac{d\lambda}{\lambda}\right) - \frac{\sum_i H_i W_i}{C}\frac{d\lambda}{\lambda} + \frac{(d\Pi + rdM)}{C}$$
$$\implies \frac{d\mathcal{L}^*}{\lambda C} = \frac{W_i}{\eta_i}\frac{dH}{C} + \Theta\frac{d\lambda}{\lambda} - \frac{\sum_i H_i W_i}{C}\frac{d\lambda}{\lambda} + \frac{(d\Pi + rdM)}{C}$$
$$\implies \frac{d\mathcal{L}^*}{\lambda C} = \frac{W_i}{\eta_i}\frac{dH}{C} + \left(\Theta - \frac{\sum_i H_i W_i}{C}\right)\gamma\left(\Theta \cdot \frac{dH}{H} - \frac{dC}{C}\right) + \frac{(d\Pi + rdM)}{C}.$$